Appendix 4

Equations[1]

Block 2. The Mortgage Model.

I. Estimation of the household net income:

$$Netinc_i = Inc_i - oblpaym_i \tag{1.1}$$

II. Calculating eligibility for subsidy

If Subs_use = 1, use subsidies.sav file, (susbs_dum$_i$ = 0, subs =0 , subsls = 0, subsir = 0)

III. Calculating intention to move

If Intmove = 1, use intention.sav file, (intmoved$_i$=1)

IV. Definition of savings which are used to buy a house (Sav)

1. Correcting available data on savings

$$Sav1_i = \frac{Inc_i}{Totinc} \cdot Savtot + Ssav_i \tag{1.2}$$

2. Determining the status of dwelling

$$Own_i = if\,(ownst_i = 1 or\, 2,1,0) \tag{1.3}$$

3. Coping "renters and splitters"

   i. Copy observations to the end of file (starting with N+1) if Own=0 or Bigfamd=1

4. Estimating money, receiving from selling a existing house

   i. Estimation dwelling value

$$Selldwpr_i = \begin{cases} if\,(own_i = 1, Esprice_i, 0), n = 1,2,...,N \\ 0, n = N+1,... \end{cases} \tag{1.4}$$

   ii. Determining available money from selling

$$Sav2_i = Selldwpr \tag{1.5}$$

5. Total amount of money that household could spend on housing purchase (including lump-sum subsidy)

$$Sav_i = Sav1_i + Sav2_i + Subsls \cdot Subs\_dum_i \tag{1.6}$$

V. Determining affordable loan size

1. Determining monthly rates for insurance

$$InsrateM = \left(1 + \frac{Insrate}{100}\right)^{\frac{1}{12}} - 1 \tag{1.7}$$

[1] In this version of the model estimations are based on the Russian statistical data. .

Typically life and disability insurance payments are made once a year. Considering that the model is designed to cover a monthly time period, it is necessary to determine the monthly rate of insurance payments. For this purpose the efficient interest rate concept should be used.

2. Determining monthly loan rate

$$r_i = \frac{lrate - subsir \cdot Subs\_dum_i}{1200} \tag{1.8}$$

3. According to C1 ratio:
   a. Monthly loan payments (Paym1):

$$Paym1_i = if(pi\_use = 0, Netinc_i \cdot C1/100, use\ separate\ file) \tag{1.9}$$

   b. Loan size

$$Loan1_i = Paym1_i \cdot \left[\frac{r}{1-(1+r)^{-t\cdot 12}} + InsrateM \right]^{-1} \tag{1.10}$$

4. Determining affordable loan size

$$Ltv1_i = Ltv + Subs \cdot Subs\_dum_i \tag{1.11}$$

$$Loan3_i = \min(Loan1_i, Loan\max \cdot 1000) \tag{1.12}$$

$$Loan4_i = \frac{Sav_i - Appl\cos t - Appr\cos t - \mathrm{Re}\, g\cos t}{\frac{Ntr\_\cos t}{100} + \frac{100+Brk\_\cos t+Tran\_cst}{Ltv1} - 1} \tag{1.13}$$

$$Loan5_i = \min(Loan3_i, Loan4_i) \tag{1.14}$$

$$Loan6_i = if(Loan5_i >= Loan\min \cdot 1000, Loan5_i, 0) \tag{1.15}$$

5. Taking in mind credit history

$$Loan7_i = if(Usehistory = 1 and\ Crhistory_i = 0, 0, Loan6_i) \tag{1.16}$$

6. Checking age limitations

$$Aged0_i = if(\exists\ 2\ persons \mid Age_i \le Age\max\ and\ Age_i \ge Age\min\ and\ relat_i = 1,2,3,4,5,6,7,8;1;0)$$

$$Aged1_i = if\left(hhsize - \sum_k (Age_i < 18) \right) = 1\ and\ Age_i >= Age\min\ and\ Age_i <= Age\max, 1, 0_i) \tag{1.17}$$

$$Loan_i = if(Aged0_i = 1\ or\ Aged1_i = 1, Loan7_i, 0) \tag{1.18}$$

7. Determining net available savings for purchase (after payments for registering a deed and other payments)

$$Sav3_i = Sav_i - Appl\cos t - Appr\cos t - \mathrm{Re}\,g\cos t - \frac{Ntr\_\cos t}{100}\cdot Loan_i - \frac{Br\,\mathrm{k}\_\cos t + Tran\_cst}{100}\cdot\frac{Loan_i}{\frac{Ltv1}{100}}$$

$$Netsav_i = if\left(Loan_i = 0, \frac{Sav_i}{1+\frac{Tran\_cst}{100}}, Sav3_i\right) \tag{1.19}$$

VI. Determining maximum available money for purchase

$$Money_i = Loan_i + Netsav_i \tag{1.20}$$

VII. Accounting the intention to move

$$If(\mathrm{int}\,move = 1\,and\,\mathrm{int}\,moved_i = 0)\,Loan_i = 0\,and\,Sav_i = 0\,Money_i = 0\,Netsav_i = 0 \tag{1.21}$$

VIII. Determining loan affordability

$$Loanaf_i = if(Loan_i = 0,0,1) \tag{1.22}$$

IX. Calculating coefficient of sample size

$$sizek = \frac{Tothous}{\sum_i weights_i}\cdot 1000 \tag{1.23}$$

X. Calculating other resulting variables

1. Share of households that can afford raising a loan

$$Shloanaf = \frac{\sum_i Loanaf_i\cdot weights_i}{\sum_i weights_i}\cdot 100\% \tag{1.24}$$

2. Share of low-income household that can afford rising a loan among all households that can afford do it.

$$Slnaf\_li = \frac{\sum_{i\in low-income} Loanaf_i\cdot weights_i}{\sum_i Loanaf_i\cdot weights_i}\cdot 100\% \tag{1.25}$$

3. Total amount of loans that can be extended

$$Totloan = \sum Loan_i\cdot weights_i\cdot Sizek \tag{1.26}$$

4. Total potential demand for housing

$$Totmoney = \sum Money_i\cdot weights_i\cdot Sizek \tag{1.27}$$

5. Average loan size

$$Loanaver = \frac{Totloan}{sizek\cdot\sum_i Loanaf_i\cdot weights_i} \tag{1.28}$$

6. Average LTV ratio

$$Ltvaver = \frac{Totloan}{sizek \cdot \sum_i \left(Loanaf_i \cdot Money_i \cdot weights_i \right)} \cdot 100 \tag{1.29}$$

7. Loan distribution statistics

i. By Income

- Volume

$$Loani1v = \sum_{20\%\, lowest\ income\ households} Loan_i \cdot weights_i \cdot Sizek \tag{1.30}$$

$$Loani2v = \sum_{40\%\, lowest\ income\ households} Loan_i \cdot weights_i \cdot Sizek - Loani1v \tag{1.31}$$

$$Loani3v = \sum_{60\%\, lowest\ income\ households} Loan_i \cdot weights_i \cdot Sizek - Loani2v - Loani1v \tag{1.32}$$

$$Loani4v = \sum_{80\%\, lowest\ income\ households} Loan_i \cdot weights_i \cdot Sizek - Loani3v - Loani2v - Loani1v \tag{1.33}$$

$$Loani5v = Totloan - Loani4v - Loani3v - Loani2v - Loani1v \tag{1.34}$$

$$Giniinv = \sum_{J=1}^{4} \left(0.2 \cdot J \cdot \sum_{X=1}^{J+1} \frac{LoaniXv}{Totloan} \right) - \sum_{J=1}^{4} \left(0.2 \cdot (J+1) \cdot \sum_{X=1}^{J} \frac{LoaniXv}{Totloan} \right) \tag{1.35}$$

- Number

$$Loani1n = \sum_{20\%\, lowest\ income\ households} Loanaf_i \cdot weights_i \cdot Sizek \tag{1.36}$$

$$Loani2n = \sum_{40\%\, lowest\ income\ households} Loanaf_i \cdot weights_i \cdot Sizek - Loani1n \tag{1.37}$$

$$Loani3n = \sum_{60\%\, lowest\ income\ households} Loanaf_i \cdot weights_i \cdot Sizek - Loani2n - Loani1n \tag{1.38}$$

$$Loani4n = \sum_{80\%\, lowest\ income\ households} Loanaf_i \cdot weights_i \cdot Sizek - Loani3n - Loani2n - Loani1n \tag{1.39}$$

$$Loani5n = \sum_i Loanaf_i \cdot weights_i \cdot Sizek - Loani4n - Loani3n - Loani2n - Loani1n \tag{1.40}$$

$$Giniinn = \sum_{J=1}^{4} \left(0.2 \cdot J \cdot \sum_{X=1}^{J+1} \frac{LoaniXn}{sizek \cdot \sum_i Loanaf_i \cdot weights_i} \right) - \sum_{J=1}^{4} \left(0.2 \cdot (J+1) \cdot \sum_{X=1}^{J} \frac{LoaniXn}{sizek \cdot \sum_i Loanaf_i \cdot weights_i} \right) \tag{1.41}$$

8. Household statistics on loan affordability

$$Hhkids_i = if\left(\sum (Age_i < 18) \geq 1; 1; 0 \right) \tag{1.42}$$

$$Hhind_i = if\left(hhsize_i = 1; 1; 0 \right) \tag{1.43}$$

$$Hhother_i = 1 - hhkids_i - hhind_i \tag{1.44}$$

$$Hkids\_sh = \frac{\sum hhkids_i \cdot loanaf_i \cdot weights_i}{\sum hhkids_i \cdot weights_i} \cdot 100\% \tag{1.45}$$

$$Hind\_sh = \frac{\sum hhind_i \cdot loanaf_i \cdot weights_i}{\sum hhind_i \cdot weights_i} \cdot 100\% \quad (1.46)$$

$$Hoth\_sh = \frac{\sum hhother_i \cdot loanaf_i \cdot weights_i}{\sum hhother_i \cdot weights_i} \cdot 100\% \quad (1.47)$$

Block 3. "Target house" analysis

1. Determining "target house" indicators:

Tgprice1 = 50th percentile of dwelling units price distribution (2.1)

Tgprice2 = 25th percentile of dwelling units price distribution (2.2)

Tgprice3 = 10th percentile of dwelling units price distribution (2.3)

Block 4. Housing affordability analysis

I. Checking "value gap"

$$Afford0_i = if\left(Selldwpr_i \cdot \frac{Valap}{100} \le (Money_i - Selldwpr_i), 1, 0\right) \quad (3.1)$$

It is presumed that the household will not conclude a deal if the value of the target house overruns the value of the house it is going to sell by an amount less than the value gap. If the household refuses to sell the available house for some reasons, this formula will show whether the value of the target house is not less the value gap, otherwise the deal will not be concluded. Determining share of households that can afford purchasing "target house 1" (median-priced home)

II. Determining share of households that can afford purchasing "target house 1" (with highest price)

1. Is "target house" affordable?

$$Dummy1_i = if\left(Money_i \ge Tgprice1, 1, 0\right) \quad (3.2)$$

$$Afford11_i = Dummy1_i \quad (3.3)$$

$$Affrod12_i = if\left(Afford0_i = 1 and\ Afford11_i = 1,1,0\right) \quad (3.4)$$

2. Determining a share of households that can afford such house

$$Aff\_sh1 = \frac{\sum Afford12_i \cdot weights_i}{\sum weights_i} \cdot 100 \quad (3.5)$$

3. Determining a share of households that can afford such house with loan

$$Af\_sh1l = \frac{\sum Afford12_i \cdot Loanaf_i \cdot weights_i}{\sum Afford12_i \cdot weights_i} \cdot 100 \quad (3.6)$$

4. Share of low-income household that can afford such house among all households that can afford do it

$$Af\_sh1li = \frac{\sum_{i \in low-income} Afford12_i \cdot weights_i}{\sum Afford12_i \cdot weights_i} \cdot 100\% \quad (3.7)$$

5. Total volume of loans raised to purchase a "target house"

$$Loanv1 = \sum \left(Loan_i \cdot Afford12_i \right) \cdot weights_i \cdot Sizek \quad (3.8)$$

6. Total volume of demand demonstrated by such households

$$Demandv1 = \sum \left(Money_i \cdot Afford12_i \right) \cdot weights_i \cdot Sizek \quad (3.9)$$

7. Average loan size by such households

$$Loanav1 = \frac{Loanv1}{\sum Afford12_i \cdot Loanaf_i \cdot weights_i} \quad (3.10)$$

8. Average LTV ratio by such households

$$Ltvaver1 = \frac{Loanv1}{\sum Money_i \cdot Afford12_i \cdot Loanaf_i \cdot weights_i} \cdot 100 \quad (3.11)$$

III. Determining share of households that can afford purchasing "target house 2"

1. Is "target house" affordable?

$$Afford21_i = if \left(Money_i \geq Tgprice2 \; and \; Dummy1_i = 0,1,0 \right) \quad (3.12)$$

$$Dummy2_i = Dummy1_i + Afford21_i \quad (3.13)$$

$$Affrod22_i = if \left(Afford0_i = 1 \; and \; Afford21_i = 1,1,0 \right) \quad (3.14)$$

2. Determining a share of households that can afford such house

$$Affrodsh2 = \frac{\sum Afford22_i \cdot weights_i}{\sum weights_i} \cdot 100 \quad (3.15)$$

3. Determining a share of households that can afford such house with loan

$$Af\_sh2l = \frac{\sum Afford22_i \cdot Loanaf_i \cdot weights_i}{\sum Afford22_i \cdot weights_i} \cdot 100 \quad (3.16)$$

4. Share of low-income household that can afford rising a loan among all households that can afford do it

$$Af\_sh2li = \frac{\sum_{i \in low-income} Afford22_i \cdot weights_i}{\sum Afford22_i \cdot weights_i} \cdot 100\% \quad (3.17)$$

5. Total volume of loans raised to purchase a "target house"

$$Loanv2 = \sum \left(Loan_i \cdot Afford12_i \right) \cdot weights_i \cdot Sizek \quad (3.18)$$

6. Total volume of demand demonstrated by such households

$$Demandv2 = \sum \left(Money_i \cdot Afford22_i \right) \cdot weights_i \cdot Sizek \tag{3.19}$$

7. Average loan size by such households

$$Loanav2 = \frac{Loanv2}{\sum Afford22_i \cdot Loanaf_i \cdot weights_i} \tag{3.20}$$

8. Average LTV ratio by such households

$$Ltvaver2 = \frac{Loanv2}{\sum Money_i \cdot Afford22_i \cdot Loanaf_i \cdot weights_i} \cdot 100 \tag{3.21}$$

IV. Determining share of households that can afford purchasing "target house 3" (with lowest price)

1. Is "target house" affordable?

$$Afford31_i = if \left(Money_i \geq Tgprice3 \, and \, Dummy2_i = 0,1,0 \right) \tag{3.22}$$

$$Dummy3_i = Dummy2_i + Afford31_i \tag{3.23}$$

$$Affrod32_i = if \left(Afford0_i = 1 and \, Afford31_i = 1,1,0 \right) \tag{3.24}$$

2. Determining a share of households that can afford such house

$$Affrodsh3 = \frac{\sum Afford32_i \cdot weights_i}{\sum weights_i} \cdot 100 \tag{3.25}$$

3. Determining a share of households that can afford such house with loan

$$Af\_sh3l = \frac{\sum Afford32_i \cdot Loanaf_i \cdot weights_i}{\sum Afford32_i \cdot weights_i} \cdot 100 \tag{3.26}$$

4. Share of low-income household that can afford rising a loan among all households that can afford do it

$$Af\_sh1li = \frac{\sum_{i \in low-income} Afford32_i \cdot weights_i}{\sum Afford32_i \cdot weights_i} \cdot 100\% \tag{3.27}$$

5. Total volume of loans raised to purchase a "target house"

$$Loanv3 = \sum \left(Loan_i \cdot Afford32_i \right) \cdot weights_i \cdot Sizek \tag{3.28}$$

6. Total volume of demand demonstrated by such households

$$Demandv3 = \sum \left(Money_i \cdot Afford32_i \right) \cdot weights_i \cdot Sizek \tag{3.29}$$

7. Average loan size by such households

$$Loanav3 = \frac{Loanv3}{\sum Afford32_i \cdot Loanaf_i \cdot weights_i} \tag{3.30}$$

8. Average LTV ratio by such households

$$Ltvaver3 = \frac{Loanv3}{\sum Money_i \cdot Afford32_i \cdot Loanaf_i \cdot weights_i} \cdot 100 \tag{3.31}$$

V. Indicators of specific household groups (J = 1, 2, 3)

$$s4in\min J = \min_{AffordJ2_i=1}(inc_i) \tag{3.32}$$

$$s4inmedJ = weighted\ \ average_{AffordJ2_i=1}(inc_i) \tag{3.33}$$

VI. Loan distribution statistics by Income

9. Volume

$$Loan41v = \sum_{20\%\ lowest\ income\ households} Loan_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek \tag{3.34}$$

$$Loan42v = \sum_{40\%\ lowest\ income\ households} Loan_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek - Loan41v \tag{3.35}$$

$$Loan43v = \sum_{60\%\ lowest\ income\ households} Loan_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek - \\ - Loan42v - Loan41v \tag{3.36}$$

$$Loan44v = \sum_{80\%\ lowest\ income\ households} Loan_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek \\ - Loan43v - Loan42v - Loan41v \tag{3.37}$$

$$Loan45v = Loanv1 + Loanv2 + Loanv3 - Loan44v - Loan43v - Loan42v - Loan41v \tag{3.38}$$

$$Gini4v = \sum_{J=1}^{4}\left(0.2 \cdot J \cdot \sum_{X=1}^{J+1}\frac{Loan4Xv}{Loanv1 + Loanv2 + Loanv3}\right) - \sum_{J=1}^{4}\left(0.2 \cdot (J+1) \cdot \sum_{X=1}^{J}\frac{Loan4Xv}{Loanv1 + Loanv2 + Loanv3}\right) \tag{3.39}$$

10. Number

$$Loan41n = \sum_{20\%\ lowest\ income\ households} \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot Loanaf_i \cdot weights_i \cdot Sizek \tag{3.40}$$

$$Loan42n = \sum_{40\%\ lowest\ income\ households} \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot Loanaf_i \cdot weights_i \cdot Sizek - Loan41n \tag{3.41}$$

$$Loan43n = \sum_{60\%\ lowest\ income\ households} \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek \cdot Loanaf_i \\ - Loan42n - Loan41n \tag{3.42}$$

$$Loan44n = \sum_{80\%\ lowest\ income\ households} \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek \cdot Loanaf_i \\ - Loan43n - Loan42n - Loan41n \tag{3.43}$$

$$Loan45n = \sum_i \left(Afford12_i + Afford22_i + Afford32_i\right) \cdot weights_i \cdot Sizek \cdot Loanaf_i \\ - Loan44n - Loan43n - Loan42n - Loan41n \tag{3.44}$$

$$Gini4n = \sum_{J=1}^{4}\left(0.2\cdot J\cdot\sum_{X=1}^{J+1}\frac{Loan4Xn}{sizek\cdot\sum_{i}\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot weights_i\cdot Loanaf_i}\right)$$
$$-\sum_{J=1}^{4}\left(0.2\cdot(J+1)\cdot\sum_{X=1}^{J}\frac{Loan4Xn}{sizek\cdot\sum_{i}\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot weights_i\cdot Loanaf_i}\right) \quad (3.45)$$

VII. Household statistics on home purchase affordability

$$Hkid\_sh4 = \frac{\sum hhkids_i\cdot\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot weights_i}{\sum hhkids_i\cdot weights_i}\cdot 100\% \quad (3.46)$$

$$Hin\_sh4 = \frac{\sum hhind_i\cdot\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot weights_i}{\sum hhind_i\cdot weights_i}\cdot 100\% \quad (3.47)$$

$$Hoth\_sh4 = \frac{\sum hhother_i\cdot\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot weights_i}{\sum hhother_i\cdot weights_i}\cdot 100\% \quad (3.48)$$

Block 5. Cost-benefit Analysis of subsidy schemes

I. Calculating budget expenditures for each household

1. Calculating present value factor for interest rate subsidy

$$PVF = \left[\sum_{k=1}^{t\cdot 12}\left(\frac{1+\frac{lrate}{1200}}{1+\frac{discont}{1200}}\right)^{k-1}\right] - \left[\left[\frac{lrate/1200}{1-\left(1+lrate/1200\right)^{-t\cdot 12}} + InsrateM\right]\cdot\sum_{k=1}^{t\cdot 12-1}\left(\left(1+\frac{discont}{1200}\right)^{-k}\cdot\sum_{j=1}^{k}\left(1+\frac{lrate}{1200}\right)^{j-1}\right)\right] \quad (4.1)$$

2. Calculating present value of interest rate subsidy for each household

$$PVsubsir_i = PVF\cdot Loan_i\cdot\frac{subsir}{1200}\cdot subs\_dum_i \quad (4.2)$$

3. Potential

$$Bud\exp 0_i = \left(\frac{subs}{100}\cdot Loan_i + subsls + PVsubsir_i\right)\cdot subs\_dum_i \quad (4.3)$$

4. Under “target home 1”

$$Bud\exp 1_i = Bud\exp 0_i\cdot Afford12_i \quad (4.4)$$

5. Under “target home 2”

$$Bud\exp 2_i = Bud\exp 0_i\cdot Afford22_i \quad (4.5)$$

6. Under “target home 3”

$$Bud\exp 3_i = Bud\exp 0_i\cdot Afford32_i \quad (4.6)$$

II. Repeating calculation without subsidy

As a main outcome there should be a file named “subs_raw” and containing following columns: “hhnumber”, “inc”, “afford12”, “afford22”, “afford32”, “0fford12”, “0fford22”, “0fford32”

III. Calculation the transition matrix (inside the “subs_raw” file)

$$trans11_i = if\left(0fford12_i \; and \; Afford12_i = 1 = 1,1,0\right) \tag{4.7}$$

$$trans21_i = if\left(0fford22_i = 1 and \; Afford12_i = 1,1,0\right) \tag{4.8}$$

$$trans31_i = if\left(0fford32_i = 1 and \; Afford12_i = 1,1,0\right) \tag{4.9}$$

$$trans01_i = if\left(0fford12_i + 0fford22_i + 0fford32_i = 0 and \; Afford12_i = 1,1,0\right) \tag{4.10}$$

$$trans22_i = if\left(0fford22_i = 1 and \; Afford22_i = 1,1,0\right) \tag{4.11}$$

$$trans32_i = if\left(0fford32_i = 1 and \; Afford22_i = 1,1,0\right) \tag{4.12}$$

$$trans02_i = if\left(0fford12_i + 0fford22_i + 0fford32_i = 0 and \; Afford22_i = 1,1,0\right) \tag{4.13}$$

$$trans33_i = if\left(0fford32_i = 1 and \; Afford32_i = 1,1,0\right) \tag{4.14}$$

$$trans03_i = if\left(0fford12_i + 0fford22_i + 0fford32_i = 0 and \; Afford32_i = 1,1,0\right) \tag{4.15}$$

$$trans00_i = if\left(0fford12_i + 0fford22_i + 0fford32_i = 0 and \; Afford12_i + Afford22_i + Afford32_i = 0,1,0\right) \tag{4.16}$$

IV. Calculating output variables

7. Total budget expenditures

I. Potential

$$Tbud\exp 0 = \sum Bud\exp 0_i \cdot weights_i \cdot Sizek \tag{4.17}$$

II. Present value of interest rate subsidy for each household

$$PVsubsr = \sum_i PVsubsir_i \cdot weights_i \cdot Sizek \tag{4.18}$$

III. Under “target home 1”

$$Tbud\exp 1 = \sum Bud\exp 1_i \cdot weights_i \cdot Sizek \tag{4.19}$$

IV. Under “target home 2”

$$Tbud\exp 2 = \sum Bud\exp 2_i \cdot weights_i \cdot Sizek \tag{4.20}$$

V. Under “target home 3”

$$TBud\exp 3 = \sum Bud\exp 3_i \cdot weights_i \cdot Sizek \tag{4.21}$$

2. Estimating benefits:

I. An increase in access for housing

$$D\_num = \frac{\sum_{i}\sum_{K=1}^{3} Trans0K_i \cdot weights_i}{\sum weights_i} \cdot 100\% \quad (4.22)$$

II. An increase in access for low-income households to housing

$$D\_numli = \frac{\sum_{i \in low-income}\sum_{K=1}^{3} Trans0K_i \cdot weights_i}{\sum weights_i} \cdot 100\% \quad (4.23)$$

$$D\_shli = \frac{\sum_{i \in low-income}\sum_{K=1}^{3} Trans0K_i \cdot weights_i}{\sum_{K=1}^{3} Trans0K_i \cdot weights_i} \cdot 100\% \quad (4.24)$$

III. An increase in affordability

$$Daf\_num = \frac{\sum_{i}\sum_{\substack{J,K=1 \\ J \neq K}}^{3} TransJK_i \cdot weights_i}{\sum weights_i} \cdot 100\% \quad (4.25)$$

IV. An increase in affordability for low-income households to housing

$$Daf\_nuli = \frac{\sum_{i \in low-income}\sum_{\substack{J,K=1 \\ J \neq K}}^{3} TransJK_i \cdot weights_i}{\sum weights_i} \cdot 100\% \quad (4.26)$$

$$Daf\_shli = \frac{\sum_{i \in low-income}\sum_{\substack{J,K=1 \\ J \neq K}}^{3} TransJK_i \cdot weights_i}{\sum_{i}\sum_{\substack{J,K=1 \\ J \neq K}}^{3} TransJK_i \cdot weights_i} \cdot 100\% \quad (4.27)$$

V. Subsidy distribution statistics by income

- Volume

$$Subsi1i = \sum_{20\%\, lowest\ income\ households} \left(Bud\exp 1_i + Bud\exp 2_i + Bud\exp 3_i\right) \cdot weights_i \cdot Sizek \quad (4.28)$$

$$Subs2i = \sum_{40\%\, lowest\ income\ households} \left(Bud\exp 1_i + Bud\exp 2_i + Bud\exp 3_i\right) \cdot weights_i \cdot Sizek - Subs1i \quad (4.29)$$

$$Subs3i = \sum_{60\%\, lowest\ income\ households} \left(Bud\exp 1_i + Bud\exp 2_i + Bud\exp 3_i\right) \cdot weights_i \cdot Sizek - Subs2i - Subs1i \quad (4.30)$$

$$Subs4i = \sum_{80\%\, lowest\ income\ households} \left(Bud\exp 1_i + Bud\exp 2_i + Bud\exp 3_i\right) \cdot weights_i \cdot Sizek - Subs3i - Subs2i - Subs1i \quad (4.31)$$

$$Subs5i = \left(Tbud\exp 1 + Tbud\exp 2 + Tbud\exp 3\right) - Subs4i - Subs3i - Subs2i - Subs1i \qquad (4.32)$$

VI. Household statistics on subsidy affordability

$$Hall\_sh5 = \frac{\sum\left(Afford12_i + Afford22_i + Afford32_i\right)\cdot Subs\_dum_i \cdot weights_i}{\sum weights_i}\cdot 100\% \qquad (4.33)$$

$$Hkid\_sh5 = \frac{\sum hhkids_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right)\cdot Subs\_dum_i \cdot weights_i}{\sum hhkids_i \cdot weights_i}\cdot 100\% \qquad (4.34)$$

$$Hin\_sh5 = \frac{\sum hhind_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right)\cdot Subs\_dum_i \cdot weights_i}{\sum hhind_i \cdot weights_i}\cdot 100\% \qquad (4.35)$$

$$Hoth\_sh5 = \frac{\sum hhother_i \cdot \left(Afford12_i + Afford22_i + Afford32_i\right)\cdot Subs\_dum_i \cdot weights_i}{\sum hhother_i \cdot weights_i}\cdot 100\% \qquad (4.36)$$